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Review of the selected problems connected with the thermal comfort analysis

Przegląd wybranych zagadnień związanych z oceną komfortu cieplnego

Abstract

Thermal comfort according to the ISO 7730 standards is defined in the following way: it is the situation of the mind condition expressing satisfaction with the thermal situation in man's environment (thermal neutrality). This definition is of the verbal character and specialists put a lot of effort to express it in the language of mathematics and physics. The work will present the bases of the numerical approach to thermal comfort and particularly the PMV (Predicted Mean Vote) parameter determining comfort conditions in the seven-level scale (or derogations from these conditions). The present work is of the overview character and the appropriate equations leading to determining PMV were taken from literature. The authors' own contribution is a computer programme in the Delphi language supporting calculating this indicator.

Keywords: *thermal comfort, Delphi language*

Streszczenie

Komfort cieplny wg standardów ISO 7730 [4] definiowany jest następująco: jest to sytuacja stanu umysłu wyrażająca satysfakcję z termicznej sytuacji w otoczeniu człowieka (neutralność cieplna). Ta definicja ma charakter werbalny i specjaliści włożyli wiele wysiłku, aby wyrazić ją językiem matematyki i fizyki. W pracy przedstawione zostaną podstawy liczbowego podejścia do problemu komfortu cieplnego, a w szczególności parametru PMV (*Predicted Mean Vote*) determinującego w siedmio-stopniowej skali warunki komfortu (lub odstępstwa od tych warunków). Prezentowana praca ma charakter przeglądowy i odpowiednie równania prowadzące do wyznaczenia PMV zaczerpnięto z literatury. Udziałem własnym autorów jest program komputerowy w języku Delphi wspomagający obliczenia tego wskaźnika.

Słowa kluczowe: *komfort cieplny, język Delphi*

1. Introduction

Thermal comfortis definedin the ISO 7730 [4] standardin thefollowingway: 'thatcondition of mind which expresses satisfaction with the thermal environment'. In other words, the primary thermal comfort condition is thermal neutrality, which means that a person feels neither too warm nor too cold. The definition of thermal comfort seems to be quite correct and comprehensible but it has no reference to the physical conditions of comfort and ways of its determination. It should be pointed out that the feeling of thermal comfort is individual and subjective, so the mathematical modeling of this problem refers to the mean values.Generally speaking, two conditions assuring the thermal control should be fulfilled. The first isa proper value of the body's core temperature (37°C) and the proper skin surface temperature (about 34°C). The second is the fulfillment of the body's energy balance [3]. The heatproduced by the metabolism should be equal to the amount of heat lost from the body. A mathematical form of a such balance will be discussed in the Chapter 2.

The most popular measure of thermal comfort is called a PMV-index (Predicted Mean Vote)[4]. The PMV-indexpredictsthe mean value of the subjective ratings of a group of people in agiven environment. Onthe PMV scale the values from -3 (cold) through -2 (cool), -1 (slightly cool), 0 (neutral), 1 (slightly warm), 2 (warm), 3 (hot) are distinguished. In enclosed rooms, halls, corridors etc.one can control the ambient physical parameters to ensure the value of PMV = 0. Such activities are not possible outside the buildings and then the PMV index is dependent on the local weather conditions.

In such conditions the human body has its own 'contribution' because the people have a very effective temperature regulatory system which ensures that the body's core temperature is kept at approximately 37°C (the changes of blood vessels diameters and sweating).

It should be pointed out, that the PMV index for enclosed spaces is decisive when the following conditions are met, i.e.:

- average room air temperature 10-30°C;
- average radiation temperature of the walls 10-40°C;
- room air velocity 0-1 [m/s];
- steam pressure in the room 0-2700 [Pa];
- the thermal insulation of the clothing of people who are in the room 0-2 clo (see next chapters).

In the situation when the PMV index is equal to 0 the part of people is dissatisfied with the temperature level. To predict the number of dissatisfied people the PPD index (*Predicted Percentage of Dissatisfied*) has been introduced, but these problems will not be discussed here.

The problems connected with PMV computations are the main subject of the paper presented.

It should be emphasized that the formulas and calculation methods presented in this workare taken from literature. So, the article is of a review nature. The author's own achievement is a computer program that calculates the PMV index for various external physical parameters.

2. Energy balance

Thermal comfort equation can be written in the form [1, 10, 8]:

$$M = H + E_C + C_{RES} + E_{RES} \tag{1}$$

All components in equation (1) have the dimension [W/m^2]. In above equation M is a metabolism, H is a loss of the heat from the body surface through the convection and radiation, E_C is a heat exchange through the evaporation on the skin surface, C_{RES} is the heat exchange resulting from the breathing process, E_{RES} is a heat exchange through the evaporation connected with the breathing. In some papers the left side of the equation is taken in the form $M - L$ where L is an external work, but the authorsdid not find an example where the value of L was assumed to be non-zero.

2.1. Metabolism

The productivity of internal heat sources [W/m^3] connected with the metabolism can change in the wide range, which is related to the temporary effort of the human body. The heat associated with the metabolism must be cast out through the skin surface, and the value of appropriate heat flux [W/m^2] corresponds to value of M in equation (1).

Generally, the metabolism is measured in Met and 1 Met=58.15 [W/m^2]. One can see, that a typical adult has an external skin surface of the order of 1.7 [m^2] and at theactivity level 1Met a heat loss is equal to about 100 [W].

The experimental data concerning the value of M (and Met) are collected in literature. For example, the following data can be found:

Lying	Met = 0.8,	M=46 [W/m^2];
Stand	Met = 1.2,	M=70 [W/m^2];
Run[15km/h]	Met = 9.5,	M=550 [W/m^2];
Playing tennis	Met = 4.0	M=232 [W/m^2].

If the human activity is variable (a typical situation) then the average value corresponding to the last hour should be assumed.It results from the some kind of the 'physiological memory'.

2.2. Heat loss *H*

In literature one can find the different formulas determining the value of H. They are similar and result from the summing of the radiant and convective components of the lost heat flux. These (rather obvious for the specialists) dependences, however, differ in temperatures entered into the equations determining value of H.

The basic equation concerning the heat loss is the following

$$H = \varepsilon C_c \frac{A_r}{A_{Du}} f_{cl} \left[\left(\frac{T_{cl} + 273}{100} \right)^4 - \left(\frac{T_r + 273}{100} \right)^4 \right] + f_{cl} \alpha_c \left(T_{cl} - T_a \right) \tag{2}$$

where:

ε – emission coefficientof skin surface (as a rule$\varepsilon = 0.95$),
C_c – Stefan-Boltzmann constant, $C_c = 5.67$ [W/m^{2}K^4],
T_r [°C] – mean radiant temperature,
T_{cl} [°C] – clothing surface temperature,
T_a [°C] – ambient temperature,
α_c [W/(m^{2}K)] – convective heat transfer coefficient between clothing and air,
f_{cl} – clothing area factor. The ratio of the surface area of the clothed body to the surface area of the naked body,
A_r [m^2] – effective radiant area of a body. Surface that exchanges radiant energy with the environment,
A_{Du} [m^2] – DuBois body surface area. The total surface area of a naked person estimated by the DuBois formula.

The formula determining value of H is not complex, but the correct assumption of temperatures T_r and T_{cl} is quite complicated. In the case of mean radiant temperature the problems occur when in the space the different heat sources with the different temperatures are acting (e.g. fireplaces, stoves, heated floor etc.). Then the following formula can be applied

$$T_r + 273 = \sqrt[4]{\sum_{i=1}^{n} \varphi_{P-i} \left(T_i + 273 \right)^4} \tag{3}$$

where:

φ_{P-i} is a configuration factor [9] between the person P and surface i (i=1, 2, … n), while $T_i$is the temperature of the surface$i$.

In this place one can recall the closure law, this means

$$\sum_{i=1}^{n} \varphi_{P-i} = 1 \tag{4}$$

Measuring the temperature of all surfaces in the room is very time consuming, and even moretime consuming is the calculation of the corresponding configuration factors. That is why the use of the mean radiant temperature is avoided, if possible [10].

In this connection the basic formula (2) is transformed in this way that in the place of T_r the others conventional temperatures are introduced. These are the operative and equivalent temperatures and (according to the literature) they are easier to measure or calculate.

It should be pointed out that for the simplest geometrical and thermal cases[10]one can find the experimental formulas determining directly the value of the mean radiant temperature. As an example, for a man sitting in a room the following experimental formula can be accepted

$$T_r = \frac{0.08(T_1 + T_2) + 0.23(T_3 + T_4) + 0.35(T_5 + T_6)}{2(0.08 + 0.23 + 0.35)} \tag{5}$$

where indexes 1, 2, 3, 4, 5, 6 correspond to the 'top, bottom, left right, front, back'. One can see that when the all temperatures are the same and equal to T then $T_r=T$. In turn, for a man standing in a room one has

$$T_r = \frac{0.18(T_1 + T_2) + 0.22(T_3 + T_4) + 0.33(T_5 + T_6)}{2(0.18 + 0.22 + 0.33)} \tag{6}$$

The problem ofthecorrect clothingsurface temperature determination is also complicated.

Of course, one can use the thermal imaging equipment (as it is available), but the exact mathematical calculations of T_{cl} are complicated. Generally speaking, the clothing surface temperature can be found on the basis of the mathematical model concerning the thermal processes proceeding in the system skin surface – air gap – fabric – environment. It is the complex so-called boundary problem described by the set of partial differential equations supplemented by the appropriate boundary conditions andit can be solved using the numerical methods (under certain simplifying assumptions one can sometimes find an analytical solution). These problems are discussed in the numerous articles, including the works of the authors of this article.

Here the papers [6, 2, 7] can be mentioned. It seems, that a such approach is too complicated as a method of the clothing surface temperature determination for the analysis of thermal comfort conditions. The best solution would be to performthe series of experiments (using the thermovision methods) and to collect the values of T_{cl} for the different types of clothing, different textiles and different external physical conditions. In literature the similar tables can be found both for the metabolism M and for the clothing's insulation *Clo* (see: next sub-chapter).

Now, the problem of the clothing area factor will be discussed. This parameter corresponds to the ratio of the surface area of the clothed body to the surface area of the naked body.

As one knows, the clothing protects a person from the losing heat. Unit of clothing insulation measure is 1 clo = 0.155 [m^{2}K/W] (it is a well known dimension of the thermal resistance). For the naked man clo = 0 and for the man in classic business suit clo ≈ 1.

To determine f_{cl} (clothing area factor) the values of I_{cl} [m^{2}K/W](clothing insulations) should be known. It is an average including uncovered parts of the body. The values of insulations for the big number of the different kinds of clothing can be found in literature.

The total resistance of clothing, that person is wearing, is calculated with the following formula

$$I_{cl} = 0,75\sum_{i=1}^{n} I_{cli} + c \tag{7}$$

where:

c=0.08 clo (0.12m^{2}K/W) – dimension of c is the same as clo). The value c=0 is also met, n is thenumber of garment components. Finally the clothing area factor is determined using the formulas:

$$\begin{aligned} f_{cl} &= 1.00 + 1.29\, I_{cl} \text{ for } I_{cl} < 0.078 \text{ [m}^2\text{K/W]} \\ f_{cl} &= 1.05 + 0.645\, I_{cl} \text{ for } I_{cl} \geq 0.078 \text{ [m}^2\text{K/W]} \end{aligned} \tag{8}$$

It should be pointed out, that in literature one can find the others formulas determining the values of the clothing area factor.

For example, the man is dressed in a shirt, panties, shorts, socks and boots. In the appropriate table one can find [1, 10, 8]:

shirt	I_{cl1}=0.19 clo
panties	I_{cl2}=0.04 clo
shorts	I_{cl3}=0.11 clo
socks	I_{cl4}=0.02 clo
boots	I_{cl5}=0.02 clo

Thesum of these values isequal to 0.38[clo] or 0.06[m^2 ^{o}C/W] and I_{cl}=0.124[m^2 ^{o}C/W], (see: equation (7)). Finally f_{cl} = 1.05 + 0.645 I_{cl}=1.13[m^2 ^{o}C/W].

In turn, in the case of person dressed in thesweater, jacket, panties, long trousers, socks, and boots:

$$\sum_{i=1}^{6} I_{cli} = 0.14[\text{m}^2\text{K/W}]$$

2.3. Heat exchange through the evaporation on the skin surface

To find the value of E_c the following empirical formula is proposed in literature [8]:

$$E_c = 3.05 \cdot 10^{-3}\left[5733 - 6.99M - p_a\right] + 0.42\left(M - 58.15\right) \tag{9}$$

where p_a[Pa] is the humidity, i.e. the partial pressure of water vapor in the air. This formula does not provide the units compatibility, but is widely cited. Probably its creators attributed the adequate dimensions for numerical coefficients appearing in (9).

Saturated vapor pressure is the pressure at which the gas and liquid are in equilibrium at a certain temperature. There is a balance between evaporation and condensation. The saturated steam pressure depends on the temperature. This value can be determined from the dependence

$$p_{sa} = 100\exp\left(18.956 - \frac{4030.15}{T_a + 235}\right) \quad [\mathrm{Pa}] \tag{10}$$

where$p_{sa}$is a saturated steam pressure, T_a is an ambient temperature.
Water vapor pressure (humidity) is the pressure exerted by the vapor in the air. The concept of relative humidity is also used, namely

$$\Phi = \frac{p_a}{p_{sa}} \tag{11}$$

The knowledge (or assumption) of relative humidity allows one to find the value of p_a

$$p_a = \Phi p_{sa} \tag{12}$$

2.4. Heat exchangeresulting from the breathing process

Convective heat transfer connected with the breathing process results from the empirical equation in the form

$$C_{RES} = 0.0014M\left(34 - T_a\right) \tag{13}$$

where T_a[□] is (as previously) is an ambient temperature. To ensure the units compliance the coefficient 0.0014 should have the dimension [1/K] (1K corresponds to 1□).

2.5. Heat exchange through the evaporation connected with the breathing.

This component of the energy balance is given by the following empirical formula

$$E_{RES} = 1.72 \cdot 10^{-5} M\left(5867 - p_a\right) \tag{14}$$

where p_a[Pa] (as previously) is the humidity, i.e. the partial pressure of water vapor in the air. One can see that in order to assurethe units compliance the coefficient 1.72 should have the dimension [1/Pa].

2.6. Other parameters

In equation (2) the clothing surface temperature T_{cl} appears. As mentioned previously the assignation of this parameter is complicated. However in the literature one can find the simplified formulas, for example:

$$T_{cl} = T_{sk} - kI_{cl}f_{cl}\left[\left(T_{cl}+273\right)^4 - \left(T_r+273\right)^4\right] - I_{cl}f_{cl}\alpha_c\left(T_{cl}-T_a\right) \tag{15}$$

where (see: (2))

$$k = \varepsilon\, C_c \frac{A_r}{A_{Du}} \approx 39.6 \cdot 10^{-9}\,[\mathrm{W/m^2K^4}] \tag{16}$$

Here C_c= 5.67*10^{-8}W/[m^{2}K^4].

Similarly, the approximate formula is used for the skin temperature

$$T_{sk} = 35.7 - 0.028M \tag{17}$$

Additionally, the heat transfer coefficient between clothing and air is determined by the empirical equation

$$\begin{aligned} &\alpha_c = 2.38(T_{cl} - T_a)^{0.25} \quad \text{for} \quad 2.38(T_{cl} - T_a)^{0.25} > 12.1\,\sqrt{v}_{ar} \\ &\alpha_c = 12.11\,\sqrt{v}_{ar} \ \text{for} \quad 2.38(T_{cl} - T_a)^{0.25} \le 12.1\,\sqrt{v}_{ar} \end{aligned} \tag{18}$$

andv_{ar} is a mean air velocity [m/s].

The equation (15) has not the analytical solution and the known numerical methods should be applied[5].

Summing up, in order to determine the PMV index, the following thermal parameters should be measured:

1. air temperature;
2. mean radiant temperature (this value should be rather calculated);
3. mean air velocity;
4. humidity.

2.7. PMV index

The PMV index is determined by the following formula

$$\mathrm{PMV} = \left[0.303\exp(-0.036M) + 0.028\right]\left(M - H - E_c - C_{RES} - E_{RES}\right) \tag{19}$$

One can see that the all components creating above equation have been defined in the previous sub-chapters. The PMV index allows one to predict the conditions of thermal comfort using the seven-points scale.

3. Brief discussion of the numerical algorithm

To obtain the results of the PMV index computations, the authorial computer program has been used. Analysis of the equation (19) presented in the previous chapter shows that the first stage of computations depends on the assumption of the type of activitywhich allows one to assign the appropriate value of M. Next, one should find the total clothing resistance. Knowledge of the I_{cl} gives a possibility of f_{cl} calculation. The mean skin surface temperature is calculated from the formula (17) – or using more complicated numerical algorithm.If the ambient temperature is known or assumed (it is easy to measure), the heat exchange by the convection associated with breathing is determined from formula (13). In turn, if the moisture p_a [Pa] is known, the heat exchange by breath evaporation is determined by formula (14), while the heat exchange by evaporation on the surface of the skin results from (9).

The most difficult part of the algorithm is to determine the temperature T_{cl} (formula (15)). In this formula the heat transfer coefficient α_c appears and its value is coupled with the temperature T_{cl}. It requires the introduction of appropriate iterative procedures. The approximate value of this parameter can be determined in the more simple way assuming the knowledge of the mean heat transfer coefficient. An additional difficulty is the proper definition of the mean radiant temperature T_r, this value in the code proposed has been '*a priori*' estimated.

4. PPD index

As it is known, the feeling of thermal comfort (corresponding to the value of the PMV index equal to zero) is subjective. Some of the human population has a high tolerance for changes in thermal conditions, for the another people the range of permissible parameters is rather narrow. So, the values of the PMV index for the conditions under consideration should be treated as a certain mean value. To predict how many people are dissatisfied in a given thermal environment, the PPD-index (Predicted Percentage of Dissatisfied) has been introduced. In the PPD-index people who vote -3, -2, +2, +3 on the PMV scale are regarded as thermally dissatisfied

$$\mathrm{PPD} = 100 - 95\exp\left[-\left(0.03353PMV^4 + 0.2179PMV^2\right)\right] \tag{20}$$

Another issue is the problem of comfort in enclosed spaces with the local additional heat sources (e.g. stoves, radiators, fireplaces). In just such a case, in order to find the PMV index the very complicated methods of the mean radiant temperature must be used. From the theoretical point of view the methods of thermal analysis for this type of objects are possible, but from the practical view-point they are too complicated and difficult to use effectively. The specialists know that changing a person's location in a heterogeneously heated room, or even changing its position (state, sitting) completely changes the co-called 'a brightness balance'. This balance allows one to find the value of the heat flowing to the person from the surrounding environment. For thesubsequent positions of the individual the computations should be performed again. Fortunately, individuals can often optimize their own thermal comfort simply by adjusting their clothing to suit the conditions, for example, by removing a jumper, rolling up shirt sleeves or alternatively putting on a jacket.

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Bibliography

[1] *ASHRAE Handbook. Fundamentals*, American Society of Heating and Air ConditioningEngineers, Atlanta 1993.

[2] Ciesielski M., Duda M., Majchrzak E., Mochnacki B.: *Numerical modeling of heat transferprocesses in the human body secured with protective clothing*, Bezpieczeństwo pracy, Środowisko, Zarządzanie, red. D. Zwolińska, Wyd. WSZOP, Katowice 2015, 161-172.

[3] Fanger P.O.: *Thermal Comfort,* McGraw-Hill Book Company, 1972.

[4] *ISO 7730*, Moderate Thermal Environments – Determination of the PMV and PPD indicesand specification of the conditions for thermal comfort, 1995.

[5] Majchrzak E., Mochnacki B.: *Metody numeryczne. Podstawy teoretyczne, aspekty praktyczne i algorytmy*, Wyd. Pol. Śląskiej, Gliwice 2004.

[6] Mochnacki B.,Majchrzak E., Duda M.: *Numerical modeling of thermal processes in the living tissue domain secured with a layer of protective clothing,* Journal of Applied Mathematics and Computational Mechanics, 13, 1, 2014, 97-102.

[7] Mochnacki B., Ciesielski M.:*Sensitivity of transient temperature field in domain of forearm insulated by protective clothing with respect to perturbations of externalboundary-heat flux,* Bulletin of the Polish Academy of Sciences, Technical Sciences, 64, 3, 2016, 591-598.

[8] Parson K.,: *Human thermal environments, the effect of hot, moderate and cold environmentsin human health, Comfort and Performance*, CRS Press, Taylor & Francis-Group, 2014.

[9] Szargut J.*: Termodynamika,* Wyd. Naukowe PWN, Warszawa 2000.

[10] *Thermal Control, INNOVA,* AIR Tech. Instruments, 2002.